

1 Short Answer

- (a) Let X be uniform on the interval $[0, 2]$, and define $Y = 2X + 1$. Find the PDF, CDF, expectation, and variance of Y .
- (b) Let X and Y have joint distribution

$$f(x, y) = \begin{cases} cxy + 1/4 & x \in [1, 2] \text{ and } y \in [0, 2] \\ 0 & \text{otherwise.} \end{cases}$$

Find the constant c . Are X and Y independent?

- (c) Let $X \sim \text{Exp}(3)$.
- (i) Find probability that $X \in [0, 1]$.
 - (ii) Let $Y = \lfloor X \rfloor$. For each $k \in \mathbb{N}$, what is the probability that $Y = k$? Write the distribution of Y in terms of one of the famous distributions; provide that distribution's name and parameters.
- (d) Let $X_i \sim \text{Exp}(\lambda_i)$ for $i = 1, \dots, n$ be mutually independent. It is a (very nice) fact that $\min(X_1, \dots, X_n) \sim \text{Exp}(\mu)$. Find μ .

2 First Exponential to Die

Let X and Y be Exponential(λ_1) and Exponential(λ_2) respectively, independent. What is

$$\mathbb{P}(\min(X, Y) = X),$$

the probability that the first of the two to die is X ?