

1 How Many Marbles?

Leanne has 6 marbles, 2 red, 2 blue, and 2 green. She picks three marbles uniformly at random without replacement. Let X denote the number of blue marbles she draws.

(a) What is $\mathbb{P}[X = 0]$, $\mathbb{P}[X = 1]$, and $\mathbb{P}[X = 2]$?

(b) What do your answers you computed in part (a) add up to?

(c) Compute $\mathbb{E}[X]$ from the definition of expectation.

- (d) Suppose we define indicators X_i , $1 \leq i \leq 3$, where X_i is the indicator variable that equals 1 if the i th marble is a blue marble and 0 otherwise. Compute $\mathbb{E}[X]$ using linearity of expectation.
- (e) Are the X_i indicators independent? Does this affect your solution to part (d)?

2 Linearity

Solve each of the following problems using linearity of expectation. Explain your methods clearly.

- (a) In an arcade, you play game A 10 times and game B 20 times. Each time you play game A , you win with probability $1/3$ (independently of the other times), and if you win you get 3 tickets (redeemable for prizes), and if you lose you get 0 tickets. Game B is similar, but you win with probability $1/5$, and if you win you get 4 tickets. What is the expected total number of tickets you receive?
- (b) A monkey types at a 26-letter keyboard with one key corresponding to each of the lower-case English letters. Each keystroke is chosen independently and uniformly at random from the 26 possibilities. If the monkey types 1 million letters, what is the expected number of times the sequence “book” appears?

3 Ball in Bins

You are throwing k balls into n bins. Let X_i be the number of balls thrown into bin i .

(a) What is $\mathbb{E}[X_i]$?

(b) What is the expected number of empty bins?

(c) Define a collision to occur when a ball lands in a nonempty bin (if there are n balls in a bin, count that as $n - 1$ collisions). What is the expected number of collisions?