CS70: Lecture 9. Outline.

1. Public Key Cryptography
2. RSA system
   2.1 Efficiency: Repeated Squaring.
   2.2 Correctness: Fermat's Theorem.
   2.3 Construction.
3. Warnings.

Isomorphisms.

Bijection:

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$f(x) = ax \pmod{m}$ if $gcd(a,m) = 1$.

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Simplified Chinese Remainder Theorem:

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Bijection between $(a \pmod{n}, b \pmod{m})$ and $x \pmod{mn}$.

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Consider $m = 5$, $n = 9$, then if $(a,b) = (3,7)$ then $x = 43 \pmod{45}$.

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Consider $m = 5$, $n = 9$, then if $(a,b) = (3,7)$ then $x = 43 \pmod{45}$.

Consider $(a',b') = (2,4)$, then $x = 22 \pmod{45}$.

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Now consider: $(a,b) + (a',b') = (0,2)$.

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What is x where $x = 0 \pmod{5}$ and $x = 2 \pmod{9}$?

Try $43 + 22 = 65$

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What is x where $x = 0 \pmod{5}$ and $x = 2 \pmod{9}$?

Try $43 + 22 = 65 = 20 \pmod{45}$.

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Isomorphism:

the actions under $\pmod{5}$, $\pmod{9}$

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Try $43 + 22 = 65 = 20 \pmod{45}$.

Is it $0 \pmod{5}$? Yes! Is it $2 \pmod{9}$? Yes!

Isomorphism:
the actions under $\pmod{5}$, $\pmod{9}$
correspond to actions in $\pmod{45}$!

Poll

$x = 5 \mod 7$ **and** $x = 5 \mod 6$
$y = 4 \mod 7$ **and** $y = 3 \mod 6$

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What's true?

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What's true?

(A) $x + y = 2 \pmod 7$
(B) $x + y = 2 \pmod 6$
(C) $xy = 3 \pmod 6$
(D) $xy = 6 \pmod 7$
(E) $x = 5 \pmod{42}$
(F) $y = 39 \pmod{42}$

Poll

$x = 5 \mod 7$ **and** $x = 5 \mod 6$
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What's true?

(A) $x + y = 2 \mod 7$
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(C) $xy = 3 \mod 6$
(D) $xy = 6 \mod 7$
(E) $x = 5 \mod 42$
(F) $y = 39 \mod 42$

All true.

Xor

Computer Science:

Xor

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1 - True

0 - False

Xor

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$1 \vee 1 = 1$

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$A \oplus B$ - Exclusive or.

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Note: Also modular addition modulo 2!

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Property: $A \oplus B \oplus B = A$.

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By cases: $1 \oplus 1 \oplus 1 = 1$.

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Cryptography ...

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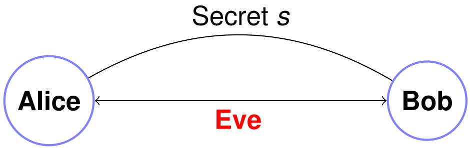

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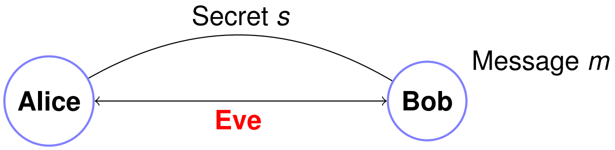

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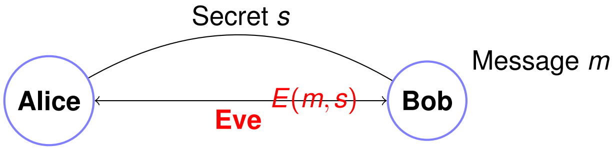

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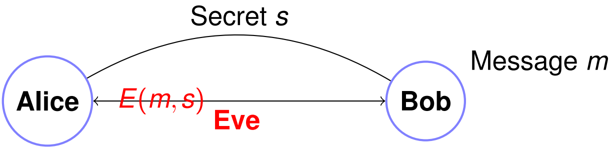

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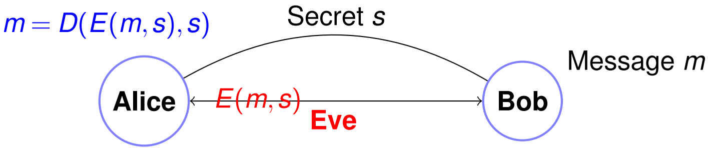
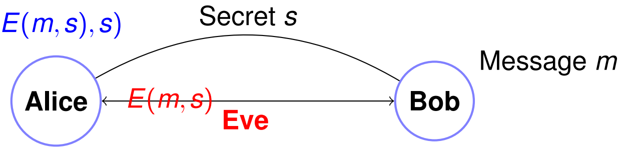


Cryptography ...

$m = D(E(m,s),s)$



Example:

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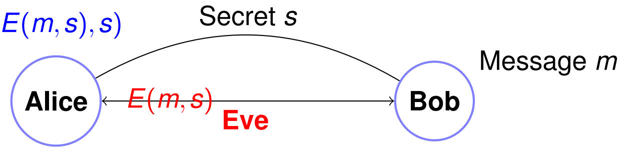

Example:

One-time Pad: secret s is string of length $|m|$.

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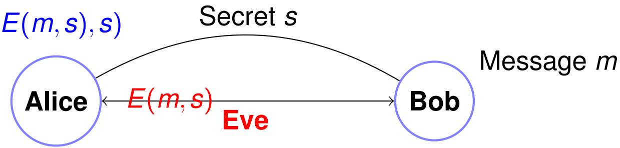

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$m = 1010101110101101$

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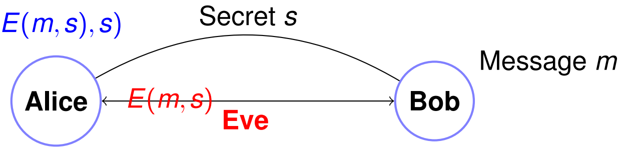

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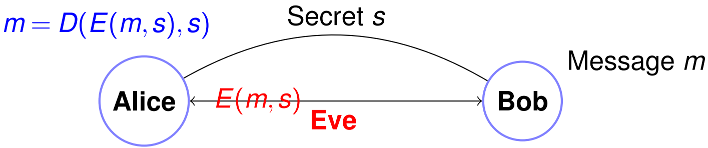

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One-time Pad: secret s is string of length $|m|$.

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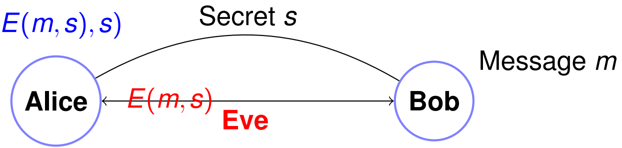
$s\ = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$E(m,s)$ – bitwise $m \oplus s$.

Cryptography ...

$m = D(E(m,s),s)$



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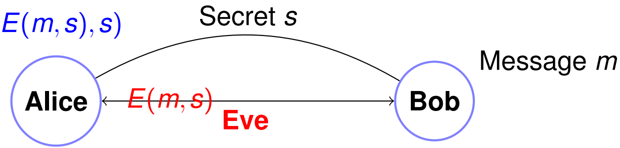

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Works because $m \oplus s \oplus s = m$!

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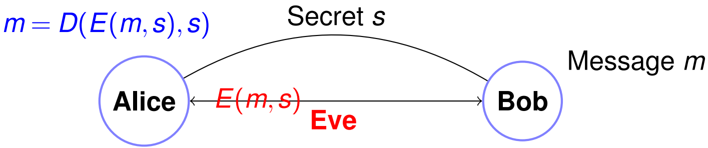

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...and totally secure!

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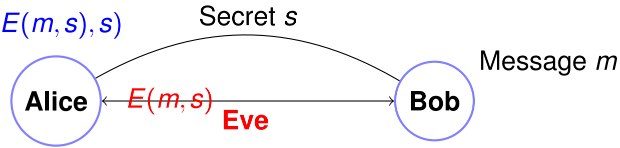

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...given $E(m,s)$ any message m is equally likely.

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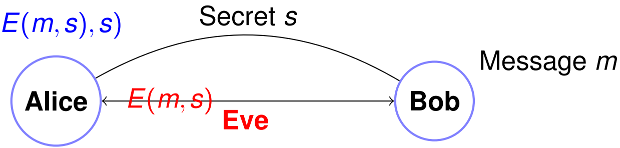

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Disadvantages:

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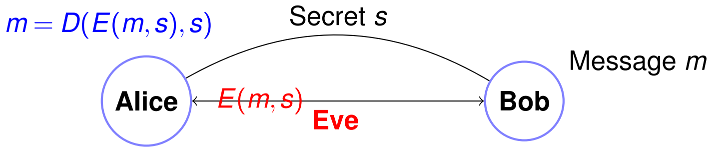

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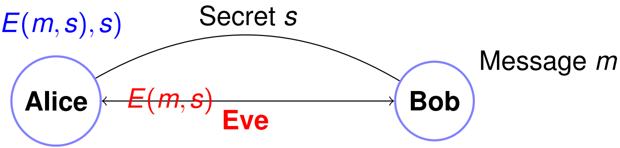
Disadvantages:

Shared secret!

Cryptography ...

$m = D(E(m,s),s)$



Example:

One-time Pad: secret s is string of length $|m|$.

$m = 1010101110101101$

$s = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$E(m,s)$ – bitwise $m \oplus s$.

$D(x,s)$ – bitwise $x \oplus s$.

Works because $m \oplus s \oplus s = m$!

...and totally secure!

...given $E(m,s)$ any message m is equally likely.

Disadvantages:

Shared secret!

Uses up one time pad..

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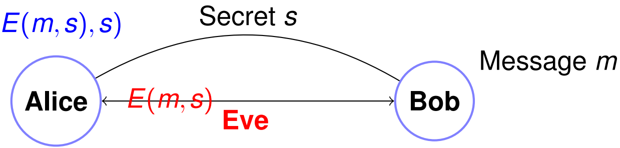

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Public key crypography.

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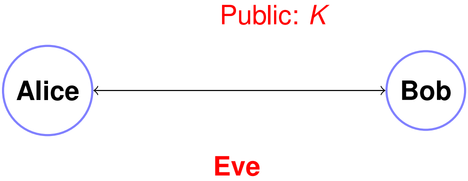

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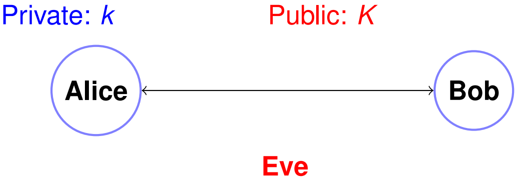

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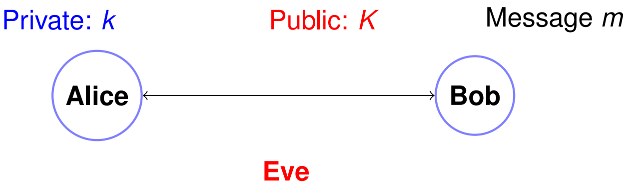

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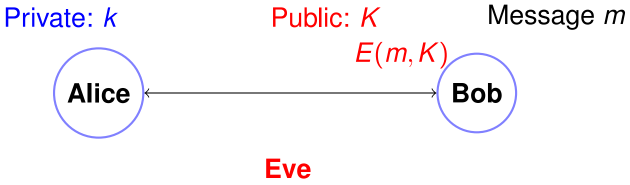

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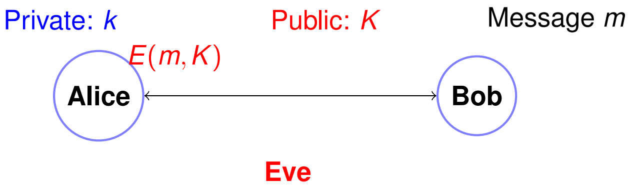

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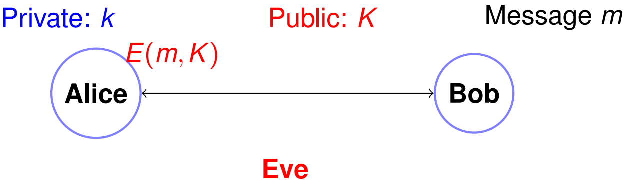

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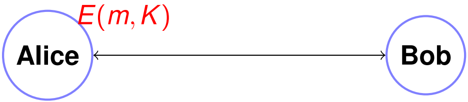
Private: k

Public: K

Message m



Eve

Everyone knows key K!

Public key crypography.

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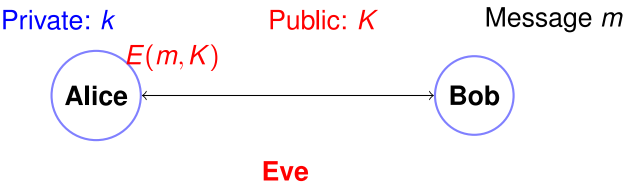

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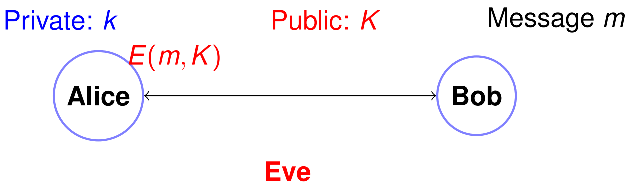

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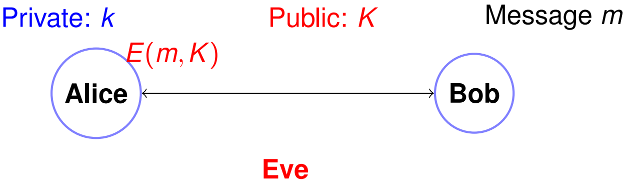

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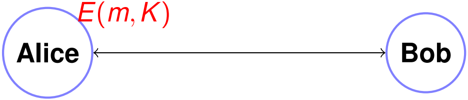

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Bob (and Eve and me and you and you ...) can encode.

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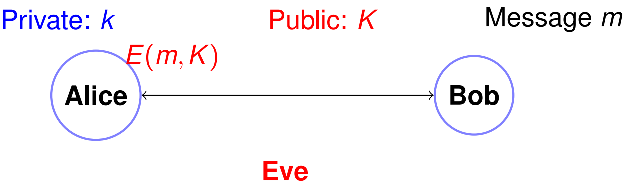

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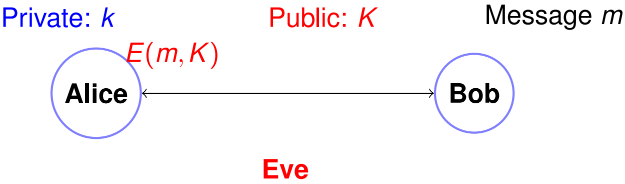

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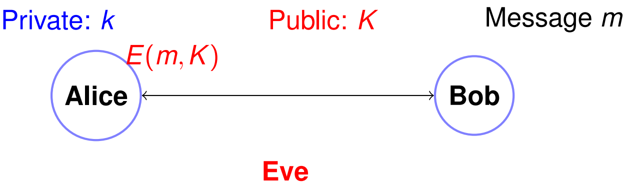

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Is this even possible?

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Example: $p = 7$, $q = 11$.

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$d = e^{-1} = -17 = 43 = \pmod{60}$

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In general, $O(N)$ or $O(2^n)$ multiplications!

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$51^{1} \equiv 51 \pmod{77}$
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Repeated Squaring took 9 multiplications versus 43.

Recursive version.

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(define (power x y m)
  (if (= y 1)
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    (let ((x-to-evened-y  (power (square x) (/ y 2) m)))
    (if (evenp y)
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Claim: Program correctly computes x^y.

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Recursive call on x^2 and $\lfloor y/2 \rfloor$, returns $(x^2)^{\lfloor y/2 \rfloor}$.

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Recursive call on x^2 and $\lfloor y/2 \rfloor$, returns $(x^2)^{\lfloor y/2 \rfloor}$.

≤ 2 multiplications per recursive call.

Note: $\lfloor y/2 \rfloor$ is integer division.

Repeated Squaring: x^y

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Repeated squaring $O(\log y)$ multiplications versus y!!!

1. x^y: Compute x^1,

Repeated Squaring: x^y

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1. x^y: Compute $x^1, x^2,$

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Repeated Squaring: x^y

Repeated squaring $O(\log y)$ multiplications versus y!!!

1. x^y: Compute $x^1, x^2, x^4, \ldots, x^{2^{\lfloor \log y \rfloor}}$.
2. Multiply together x^i where the $(\log(i))$th bit of y (in binary) is 1.

Repeated Squaring: x^y

Repeated squaring $O(\log y)$ multiplications versus y!!!

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Modular Exponentiation: $x^y \mod N$.

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Conclusion: $x^y \mod N$

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Conclusion: $x^y \mod N$ takes $O(n^3)$ time.

RSA is pretty fast.

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$D(m,(N,d)) = m^d \pmod N$.

$N = pq$ and $d = e^{-1} \pmod{(p-1)(q-1)}$.

Want: $(m^e)^d = m^{ed} = m \pmod N$.

Another view:
$d = e^{-1} \pmod{(p-1)(q-1)} \iff ed = k(p-1)(q-1)+1.$

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Similar, not same, but useful.

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From CRT: $y = x \pmod{p}$ and $y = x \pmod{q} \implies y = x$.

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All steps are polynomial in $O(\log N)$, the number of bits.

Security of RSA.

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Breaking in general sense $\implies$ factoring algorithm.

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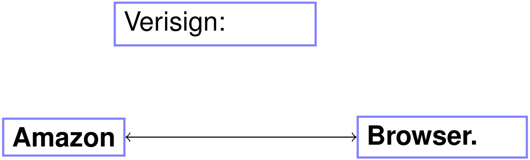
CS161...

Signatures using RSA.



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Amazon $\longleftrightarrow$ **Browser.**

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Security: Eve can't forge unless she "breaks" RSA scheme.

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Signature authority has public key (N,e).

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RSA Scheme:

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