Lecture Today.

To homework (score) or not to homework (score)

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Do proofs of optimality/pessimality again.

Lecture Today.

To homework (score) or not to homework (score)

Do proofs of optimality/pessimality again.

Graphs

Poll.

Thoughts on homework or non-homework option?

(A) Thinking about it.
(B) Definitely doing homework for score.
(C) Definitely going for the non-scored homework.

Job Propose and Candidate Reject is optimal!

For jobs?

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For jobs? For candidates?

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Theorem: Job Propose and Reject produces a job-optimal pairing.

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Assume not:

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Rogue couple for S.

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Used Well-Ordering principle...Induction.

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Structural statement: Job optimality

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Structural statement: Job optimality $\implies$ Candidate pessimality.

Lecture 5: Graphs.

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Graphs!
  Definitions: model.

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Planar graphs.

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- Graphs!
  - Definitions: model.
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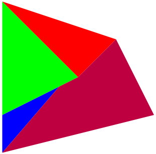
- Planar graphs.
  - Euler Again!!!!

Map Coloring.



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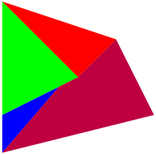

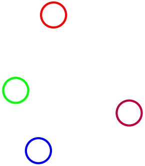

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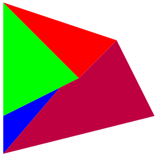

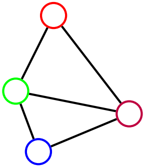

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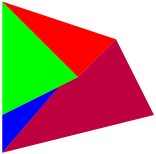
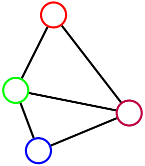
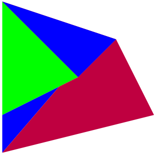
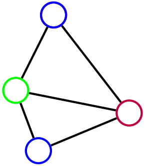




Fewer Colors?

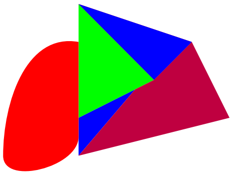
Map Coloring.

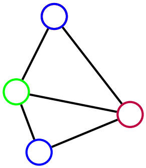




Yes! Three colors.

Map Coloring.





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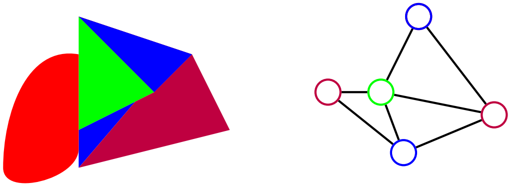

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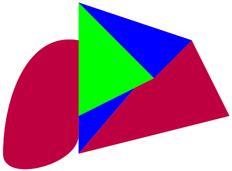

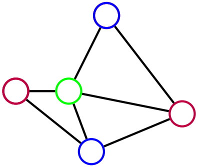

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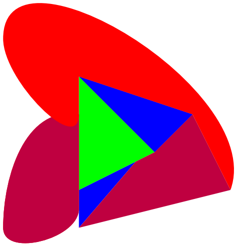

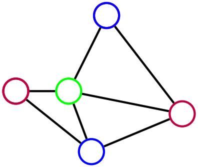

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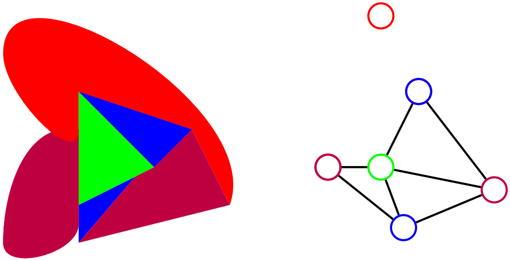

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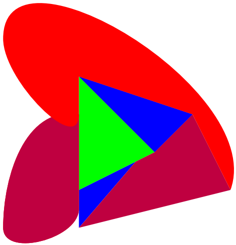

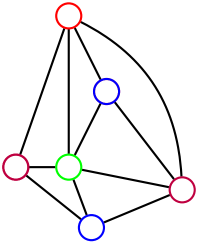

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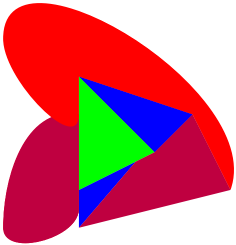

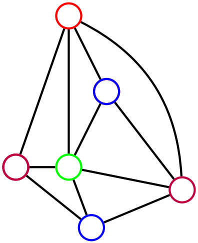

Fewer Colors?

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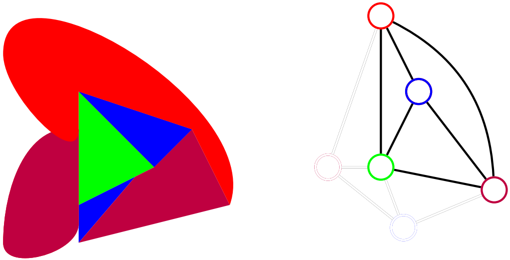

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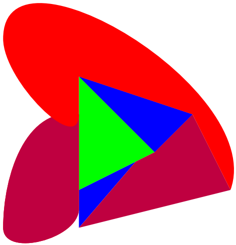

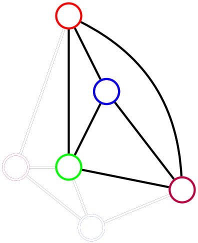

Four colors required!

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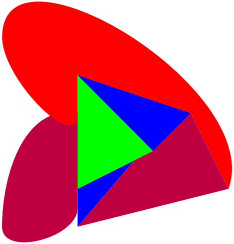
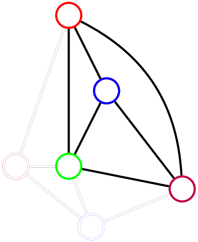
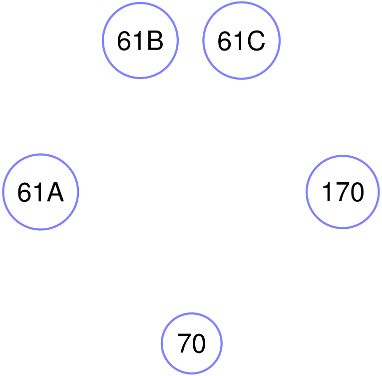




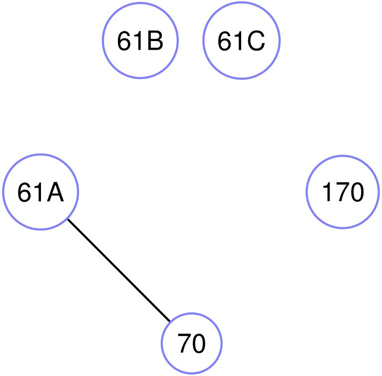
Four colors required!

Theorem: Four colors enough.

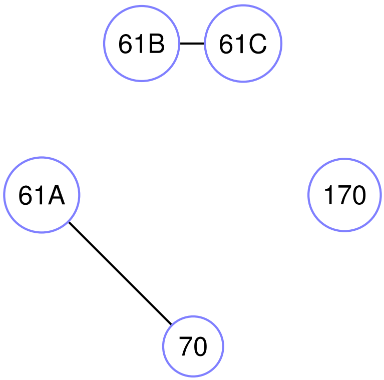
Scheduling: coloring.



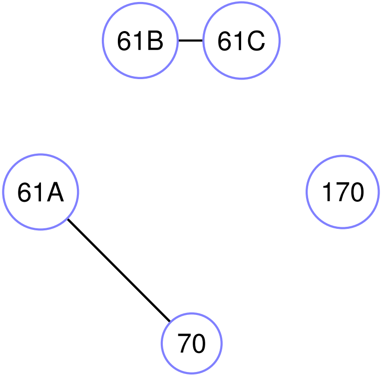
Scheduling: coloring.



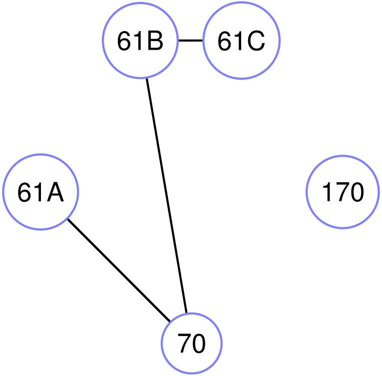
Scheduling: coloring.



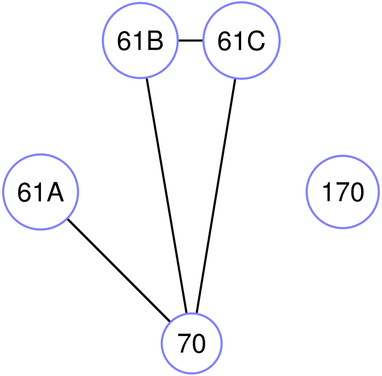
Scheduling: coloring.



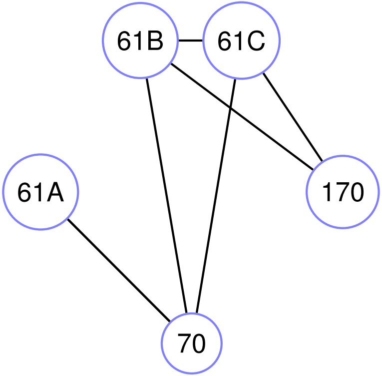
Scheduling: coloring.



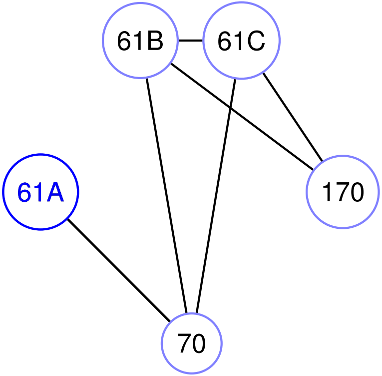
Scheduling: coloring.



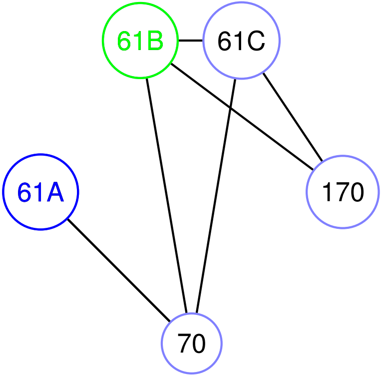
Scheduling: coloring.



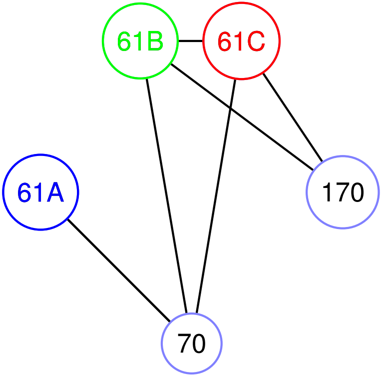
Scheduling: coloring.



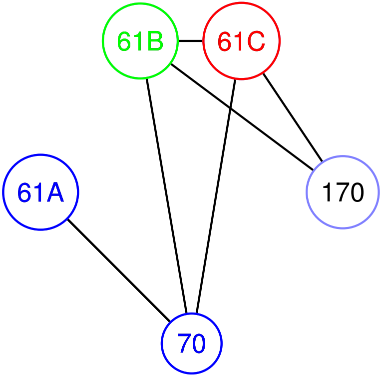
Scheduling: coloring.



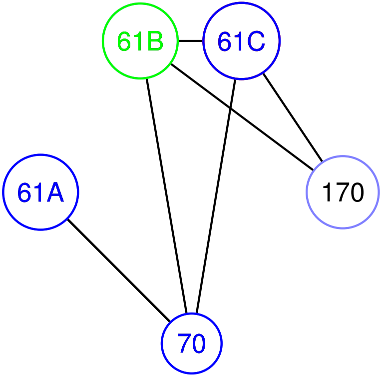
Scheduling: coloring.



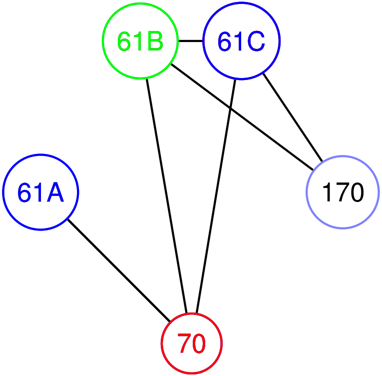
Scheduling: coloring.



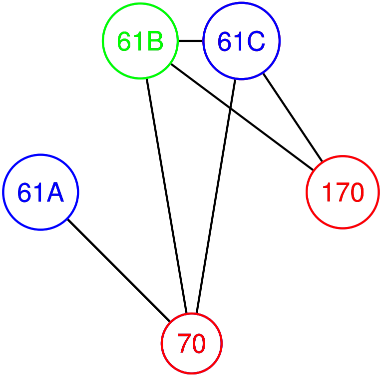
Scheduling: coloring.



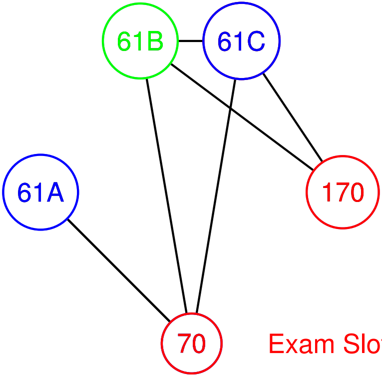
Scheduling: coloring.



Scheduling: coloring.



Scheduling: coloring.

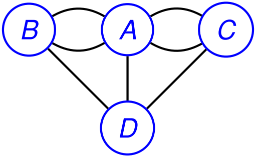


Exam Slot 1.

Exam Slot 2.

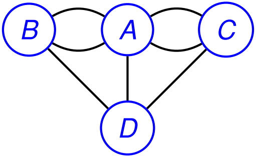
Exam Slot 3.

Graphs: formally.



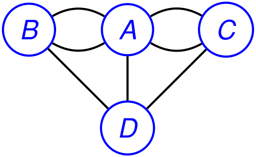
Graph:

Graphs: formally.



Graph: $G = (V, E)$.

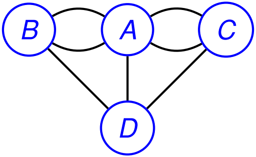
Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

Graphs: formally.

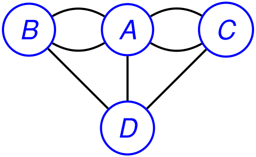


Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

Graphs: formally.



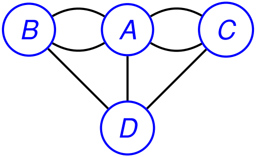
Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ -

Graphs: formally.



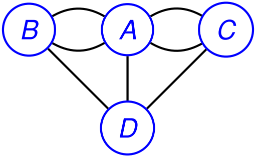
Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

Graphs: formally.



Graph: $G = (V, E)$.

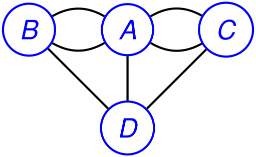
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

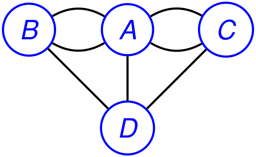
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

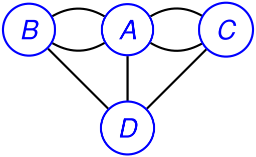
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\},$

Graphs: formally.



Graph: $G = (V, E)$.

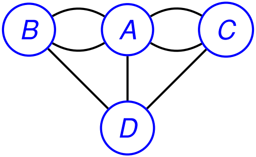
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

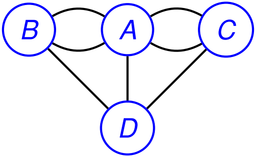
$\{A, B, C, D\}$

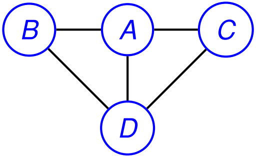
$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

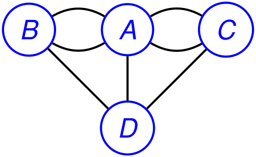
$E \subseteq V \times V$ - set of edges.

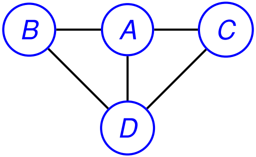
$\{\{A,B\},\{A,B\},\{A,C\},\{A,C\},\{B,D\},\{A,D\},\{C,D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

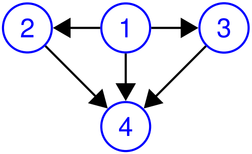
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

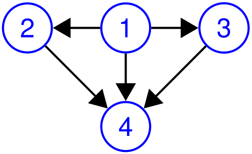
Multigraph above.

Directed Graphs



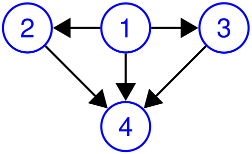
$G = (V, E)$.

Directed Graphs



$G = (V, E)$.
V - set of vertices.

Directed Graphs

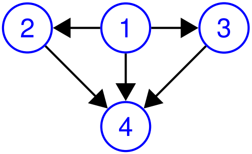


$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

Directed Graphs



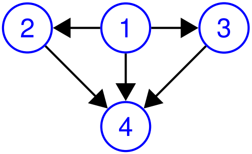
$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

Directed Graphs



$G = (V, E)$.

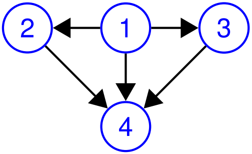
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

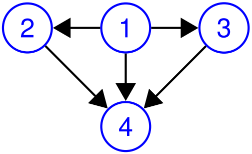
$\{(1, 2),$

Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1, 2), (1, 3),$

Directed Graphs



$G = (V, E)$.

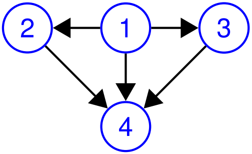
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1, 2), (1, 3), (1, 4),$

Directed Graphs



$G = (V, E)$.

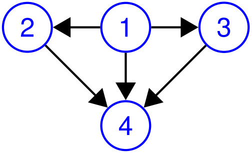
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

Directed Graphs



$G = (V, E)$.

V - set of vertices.

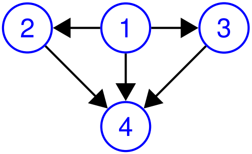
$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

One way streets.

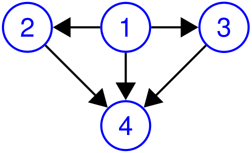
Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament:

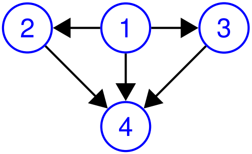
Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2,

Directed Graphs



$G = (V, E)$.

V - set of vertices.

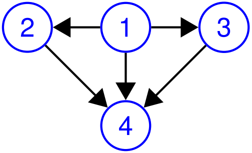
$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence:

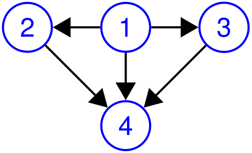
Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1,2,3,4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2,

Directed Graphs



$G = (V, E)$.

V - set of vertices.

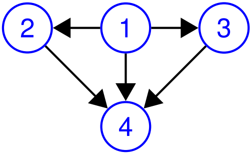
$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Directed Graphs

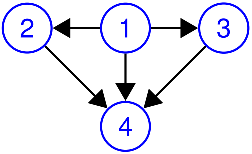


$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network:

Directed Graphs

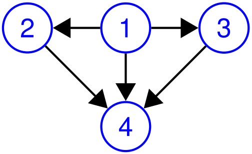


$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed?

Directed Graphs

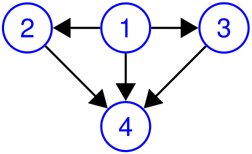


$G = (V, E)$.
V - set of vertices.
$\{1,2,3,4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?

Directed Graphs

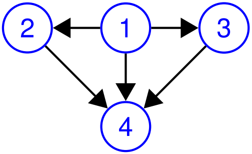


$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends.

Directed Graphs

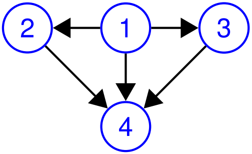


$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.

Directed Graphs

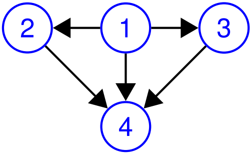


$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes.

Directed Graphs

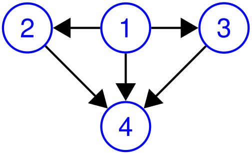


$G = (V, E)$.
V - set of vertices.
$\{1,2,3,4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes. Directed.

Directed Graphs



$G = (V, E)$.
V - set of vertices.
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E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes. Directed.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

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neighbors, adjacent, degree, incident, in-degree, out-degree

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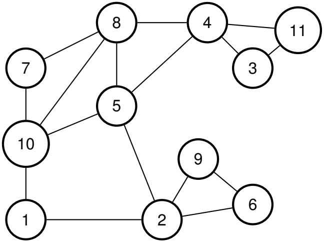

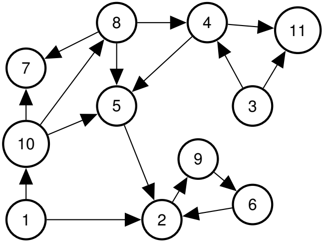

Neighbors of 10?

Graph Concepts and Definitions.

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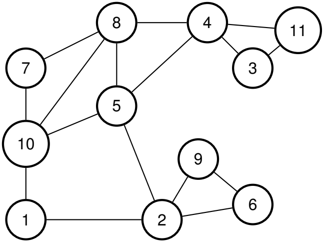

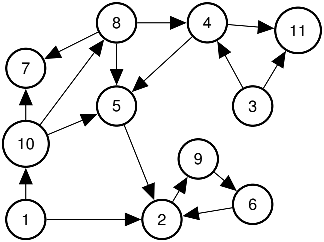

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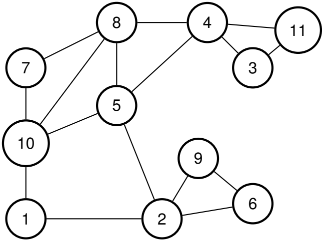

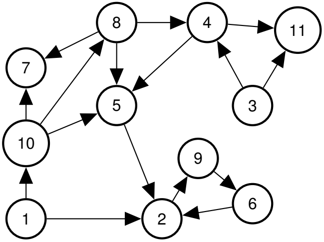

Neighbors of 10? 1,5,

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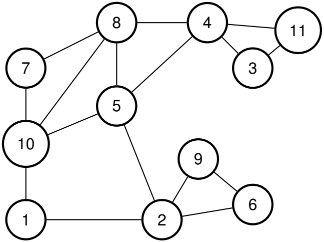

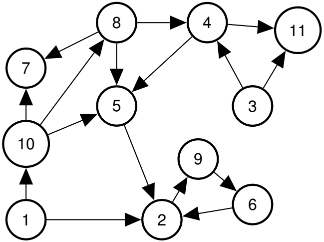

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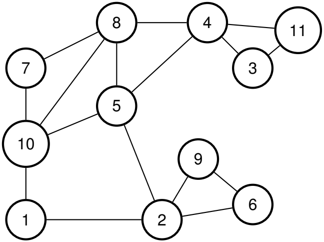

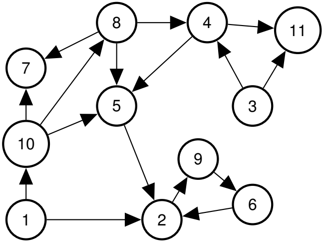

Neighbors of 10? 1,5,7, 8.

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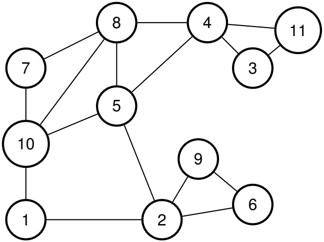

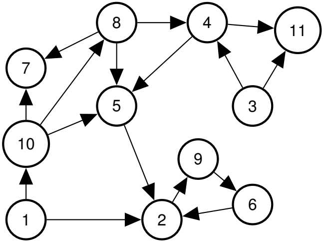

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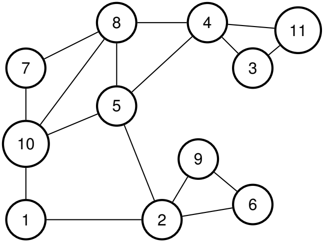

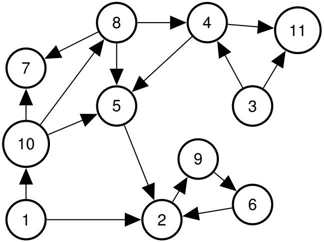

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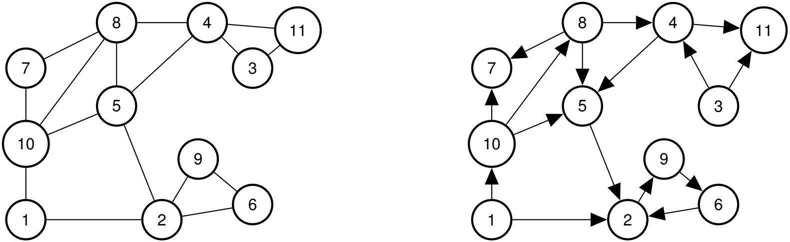

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Degree of vertex 1?

Graph Concepts and Definitions.

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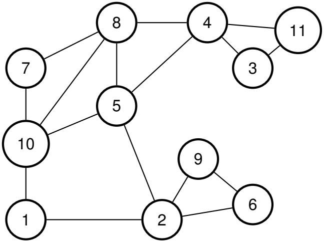

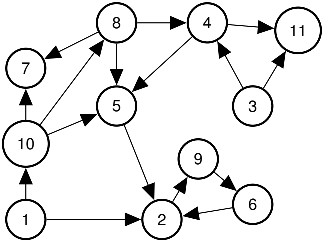

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Degree of vertex 1? 2

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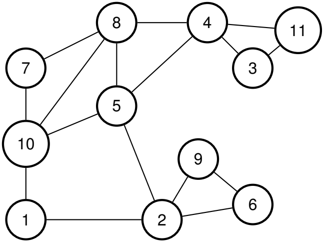

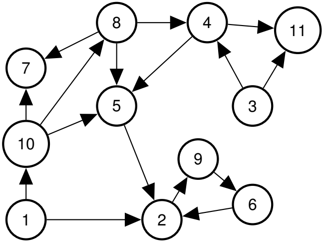

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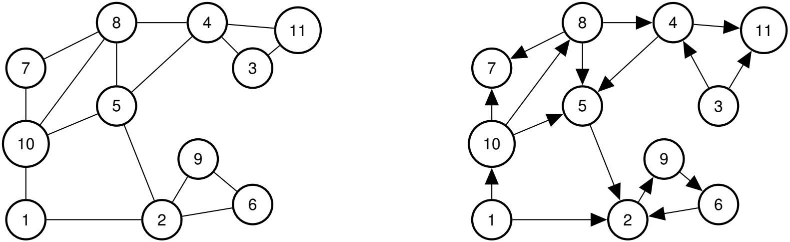

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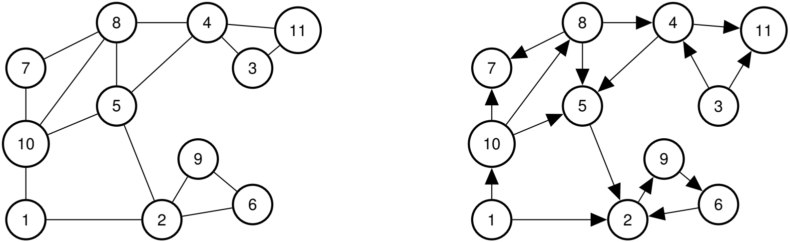

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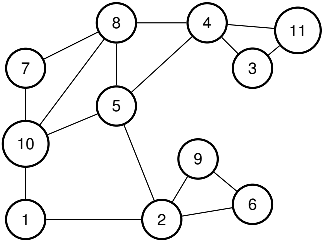

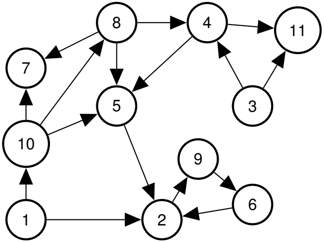

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Directed graph?

Graph Concepts and Definitions.

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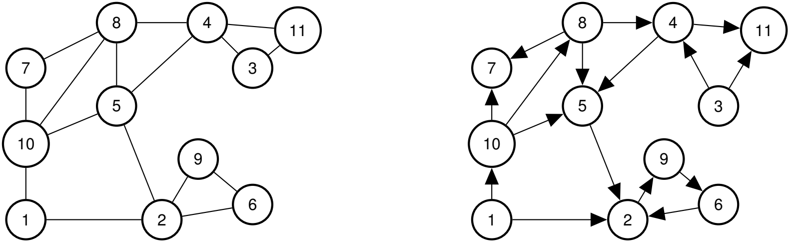

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Directed graph?

In-degree of 10?

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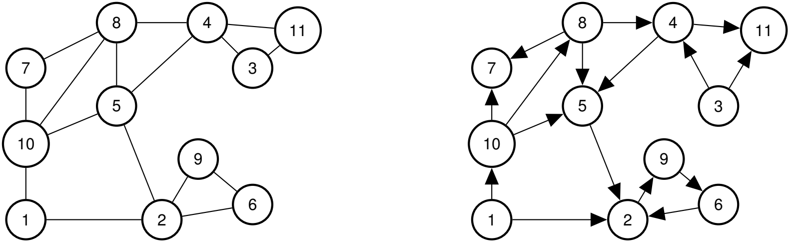
neighbors, adjacent, degree, incident, in-degree, out-degree



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Directed graph?

In-degree of 10? 1

Graph Concepts and Definitions.

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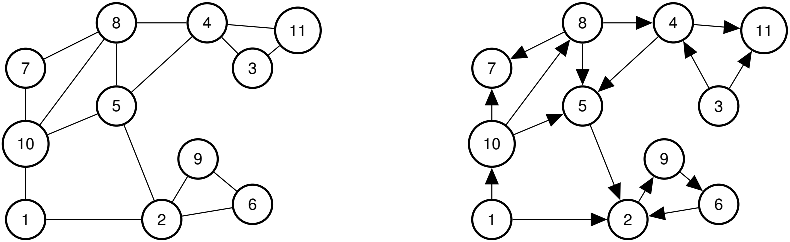

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In-degree of 10? 1   Out-degree of 10?

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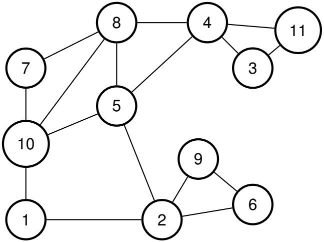

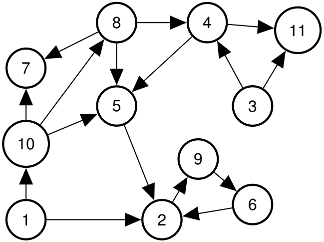

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Directed graph?

In-degree of 10? 1   Out-degree of 10? 3

Graph Concepts and Definitions.

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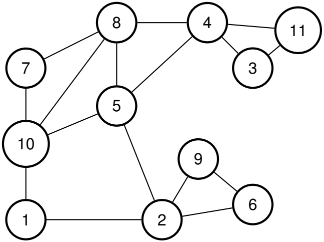

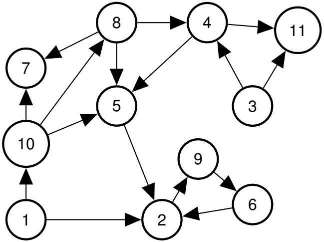

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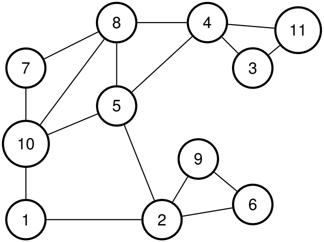

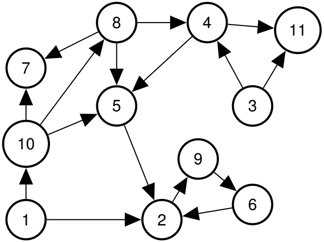

Edge $(8,5)$ **is incident to:**

(A) Vertex 8.
(B) Vertex 5.
(C) Edge $(8,5)$.
(D) Edge $(8,4)$.
(E) Vertex 10.

Graph Concepts and Definitions.

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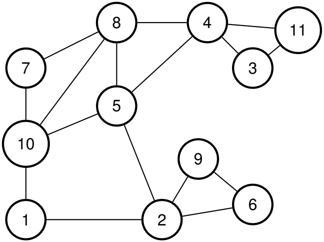

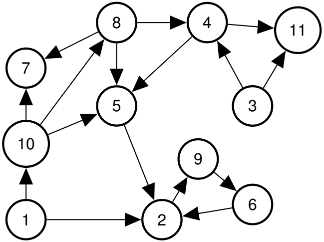

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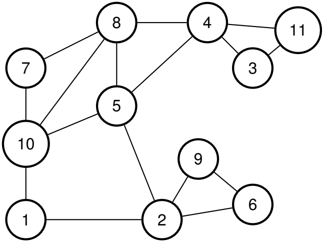

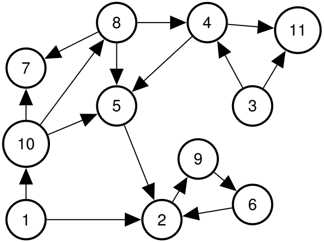

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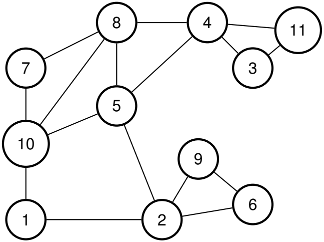

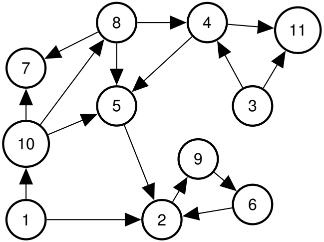

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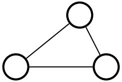
Not (A)!



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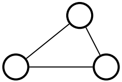

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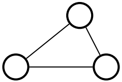

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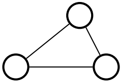

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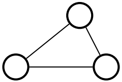

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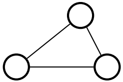

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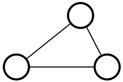

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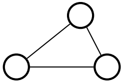
What? For triangle number of edges is 3, the sum of degrees is 6.

Sum of degrees?

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

(A) and (B) are false. (C) is a fine response to a poll with no correct answers.



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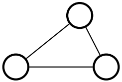

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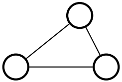

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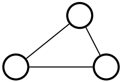

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Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

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Quick Proof of an Equality.

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edge, (u, v), is incident to endpoints, u and v.

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Let's count incidences in two ways.

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Total Incidences? The sum over vertices of degrees!

Thm: Sum of vertex degress is $2|E|$.

Poll: Proof of “handshake” lemma.

What’s true?

(A) The number of edge-vertex incidences for an edge e is 2.
(B) The total number of edge-vertex incidences is $|V|$.
(C) The total number of edge-vertex incidences is $2|E|$.
(D) The number of edge-vertex incidences for a vertex v is its degree.
(E) The sum of degrees is $2|E|$.
(F) The total number of edge-vertex incidences is the sum of the degrees.

Poll: Proof of “handshake” lemma.

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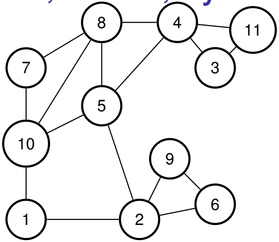
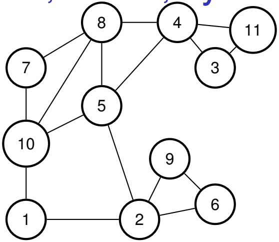
(A),(C), (D), (E), and (F).

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Paths, walks, cycles, tour.



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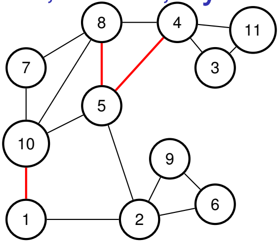

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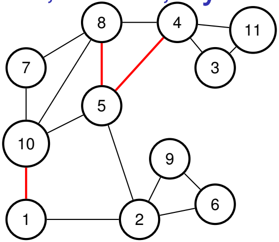
Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$?

Paths, walks, cycles, tour.



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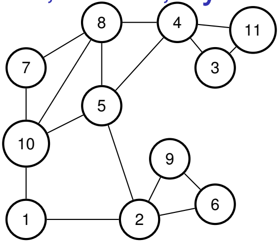

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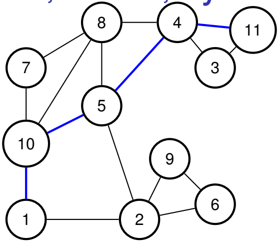

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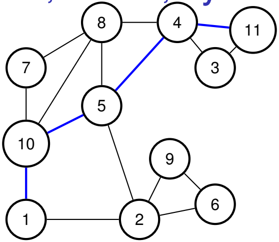

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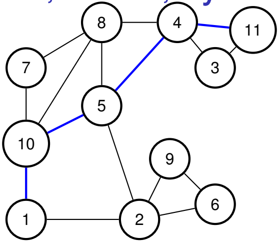

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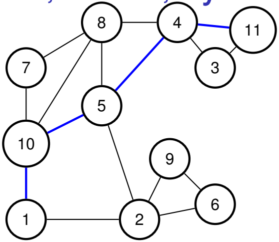

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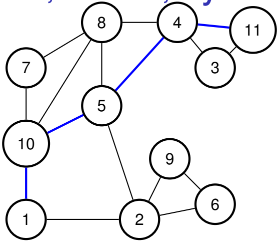
Quick Check!

Paths, walks, cycles, tour.



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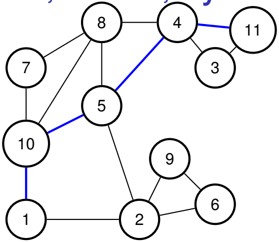
Quick Check! Length of path?

Paths, walks, cycles, tour.



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Quick Check! Length of path? k vertices

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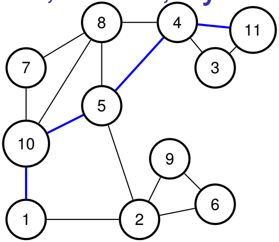

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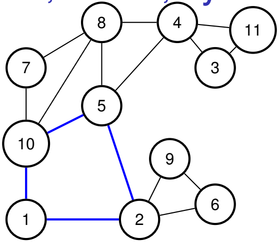

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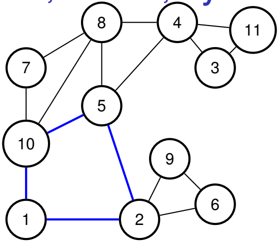
Paths, walks, cycles, tour.



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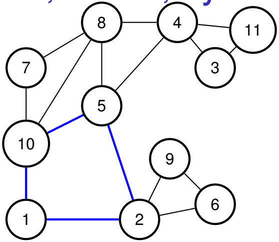

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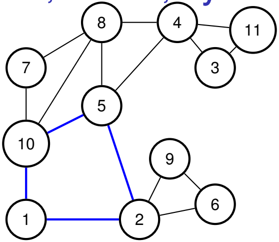

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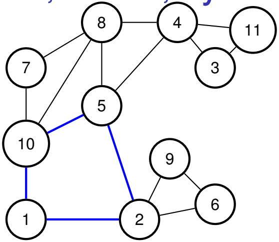
Paths, walks, cycles, tour.



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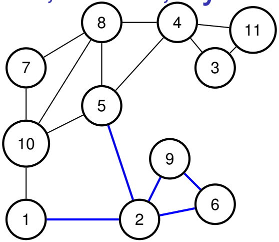
Paths, walks, cycles, tour.



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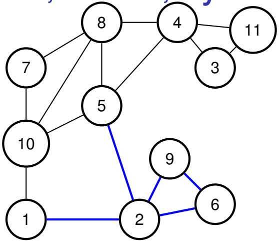
Walk is sequence of edges with possible repeated vertex or edge.

Paths, walks, cycles, tour.



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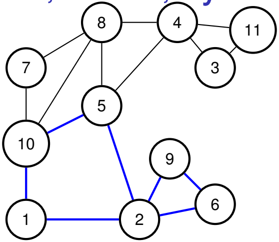

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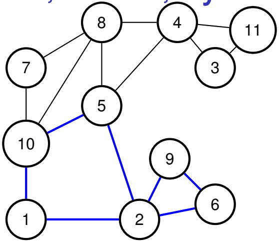
Tour is walk that starts and ends at the same node.

Paths, walks, cycles, tour.



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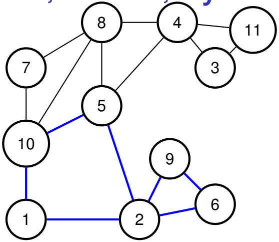

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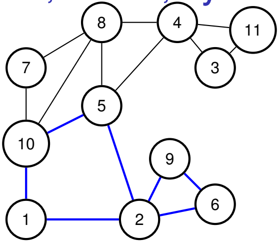

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Path is usually simple. No repeated vertex!

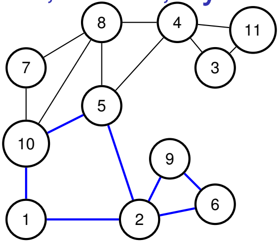
Walk is sequence of edges with possible repeated vertex or edge.

Tour is walk that starts and ends at the same node.

Quick Check!

Path is to Walk as Cycle is to ??

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Path? $\{1,10\}, \{8,5\}, \{4,5\}$? No!

Path? $\{1,10\}, \{10,5\}, \{5,4\}, \{4,11\}$? Yes!

Path: $(v_1,v_2),(v_2,v_3),\ldots(v_{k-1},v_k)$.

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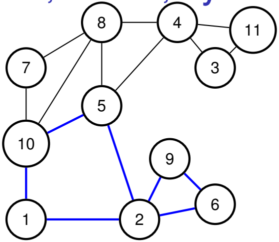

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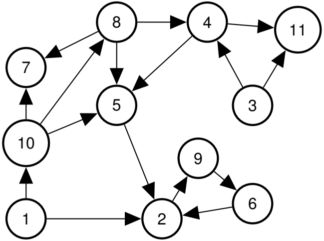
Directed Paths.



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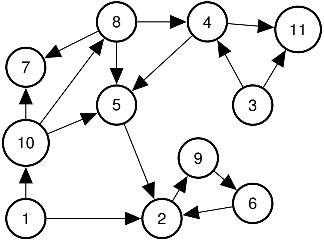

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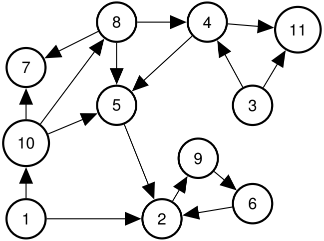

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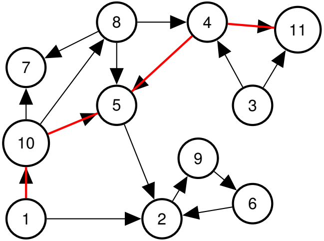

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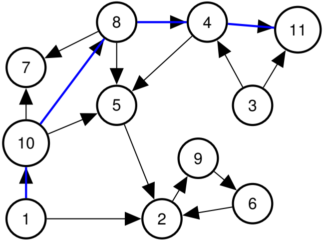

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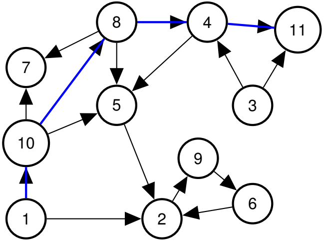

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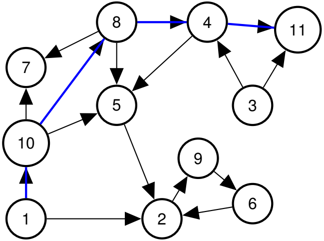

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Paths, walks,

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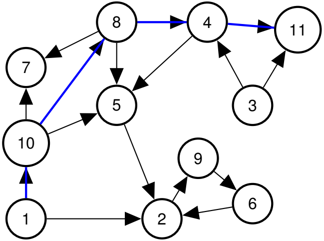

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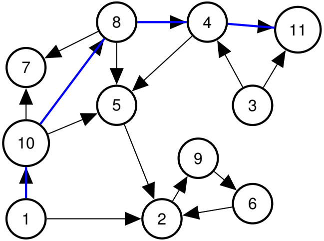

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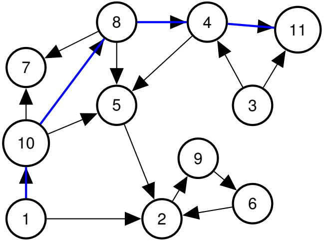
Paths, walks, cycles, tours

Directed Paths.



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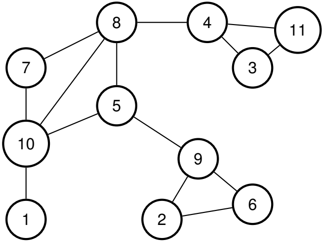
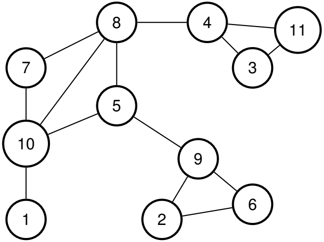
Paths, walks, cycles, tours ... are analagous to undirected now.

Connectivity: undirected graph.



u and v are connected if there is a path between u and v.

Connectivity: undirected graph.



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A connected graph is a graph where all pairs of vertices are connected.

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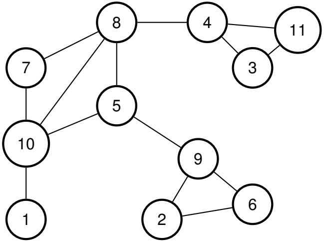

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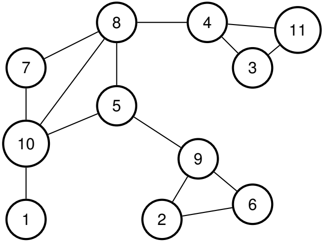

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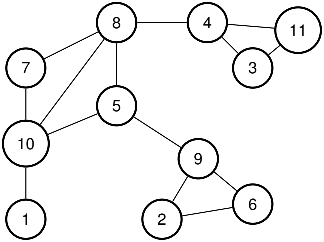

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Is graph connected? Yes?

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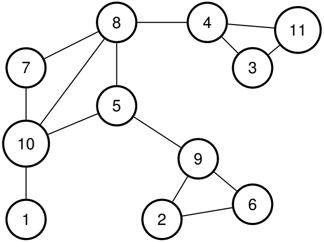

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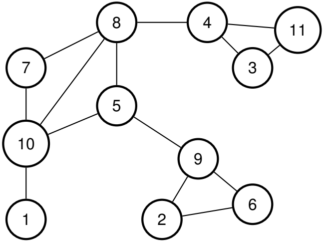

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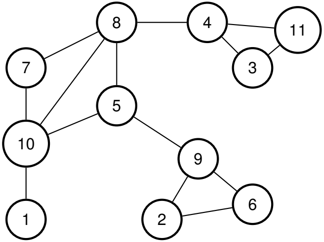

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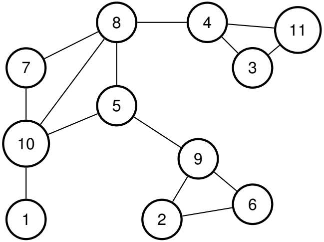

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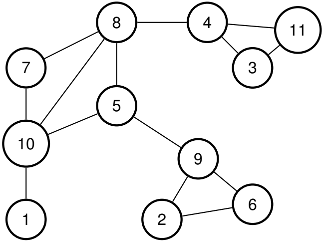

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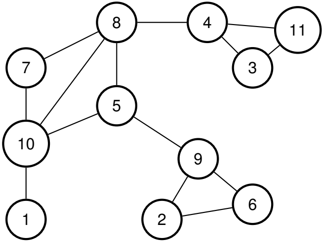

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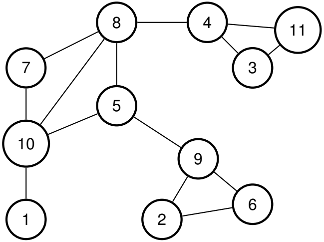

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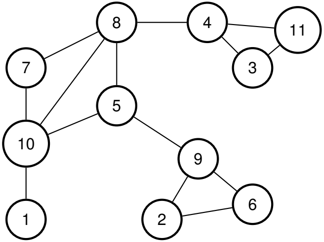

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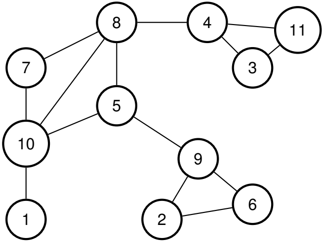

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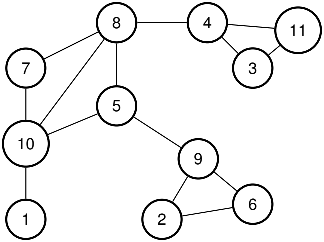
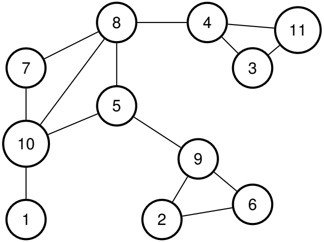


Is graph above connected?

Connected Components: Quiz.



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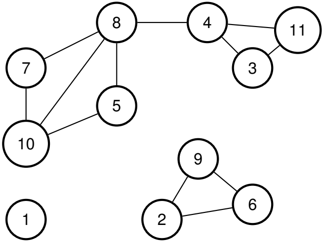

Is graph above connected? Yes!

How about now?

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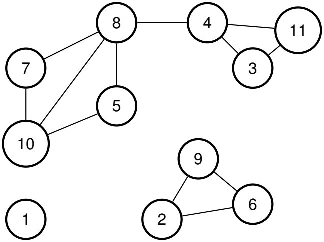

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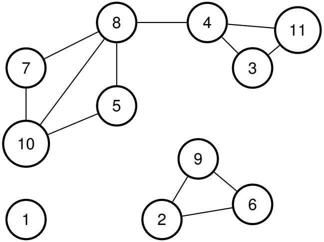

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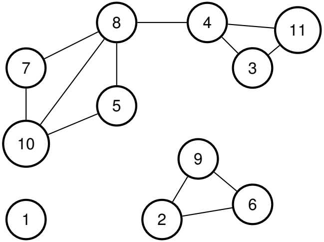
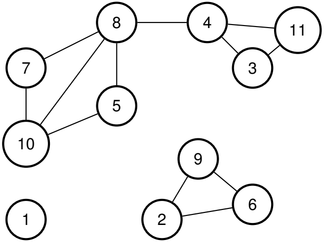


Is graph above connected? Yes!

How about now? No!

Connected Components? $\{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}$.

Connected Components: Quiz.



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Connected component - maximal set of connected vertices.

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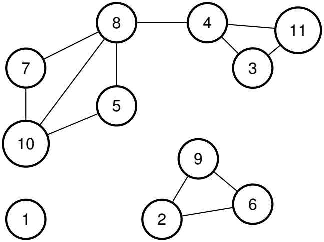

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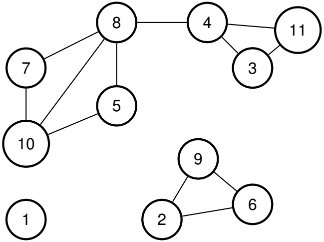
Quick Check: Is $\{10,7,5\}$ a connected component?

Connected Components: Quiz.



Is graph above connected? Yes!

How about now? No!

Connected Components? $\{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}$.
Connected component - maximal set of connected vertices.
Quick Check: Is $\{10, 7, 5\}$ a connected component? No.

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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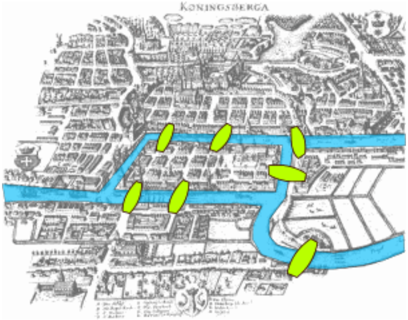

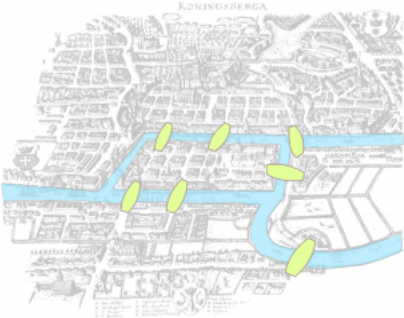

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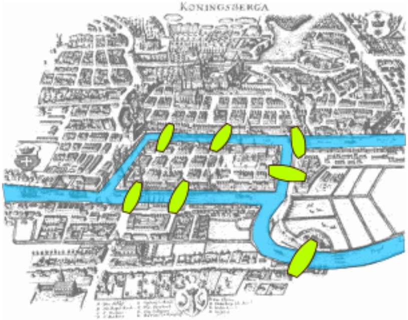

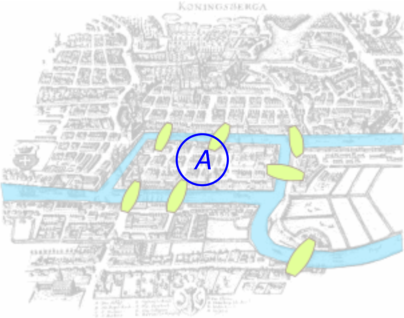

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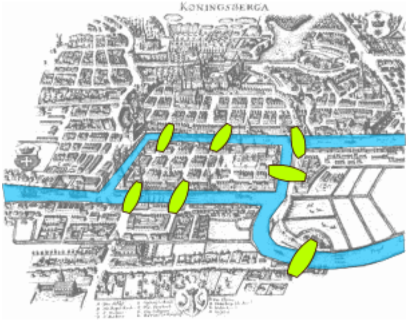

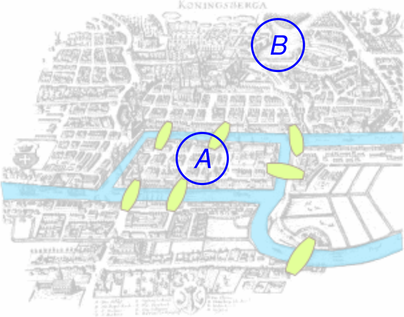

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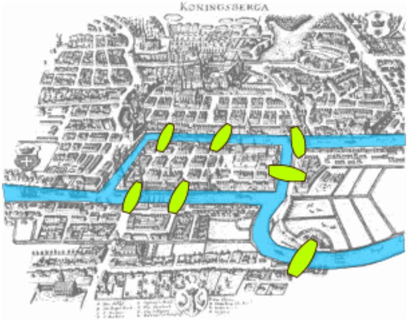

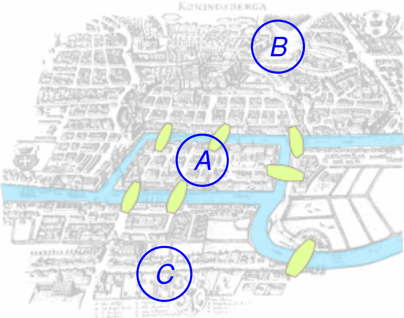

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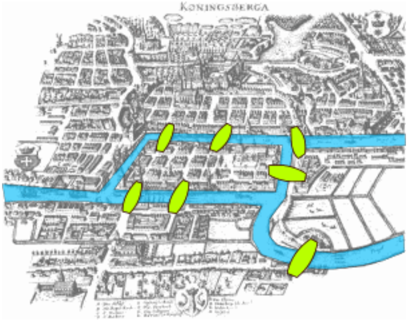

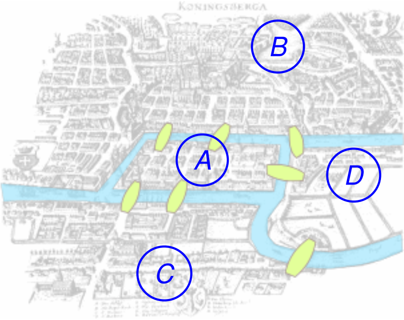

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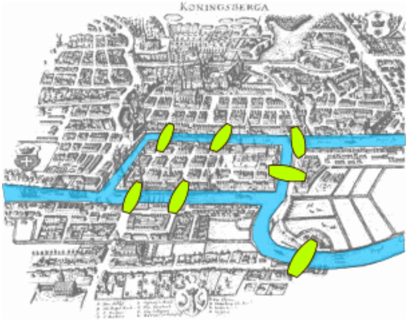

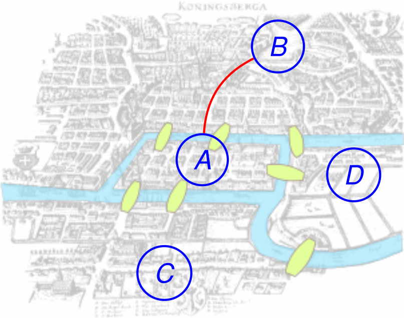

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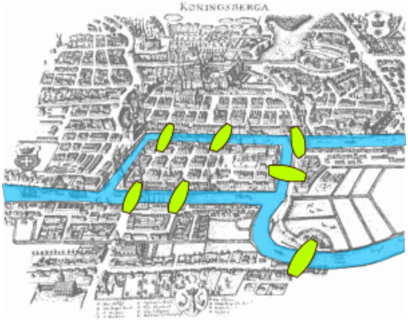

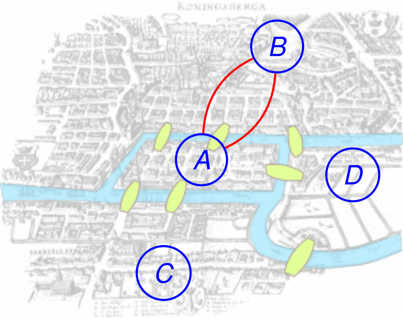

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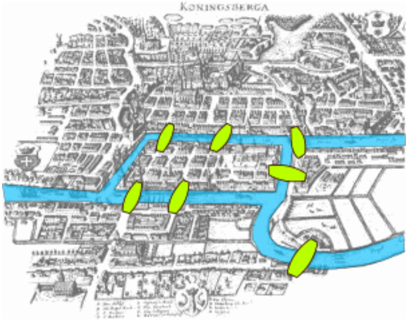
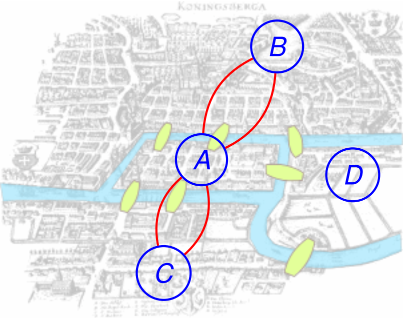
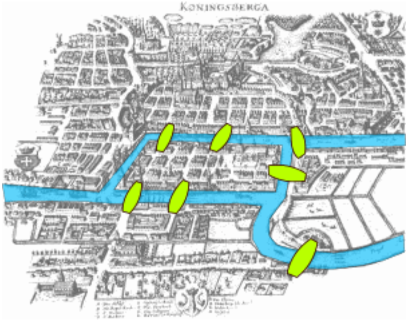


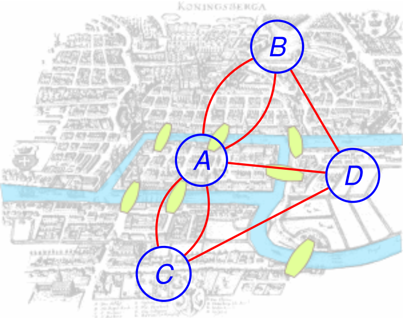


Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

“Konigsberg bridges” by Bogdan Giuşcă - License.





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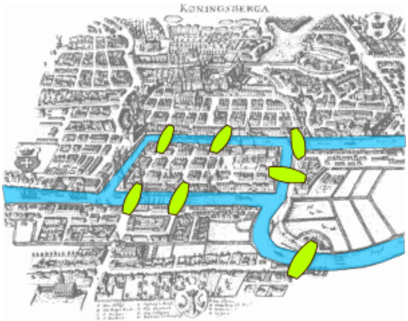

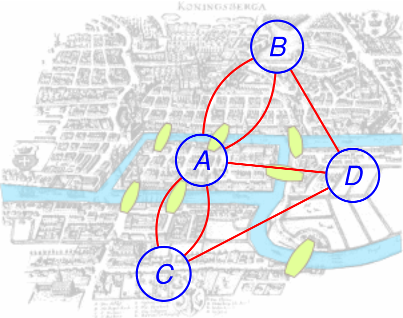

Can you draw a tour in the graph where you visit each edge once?

Konigsberg bridges problem.

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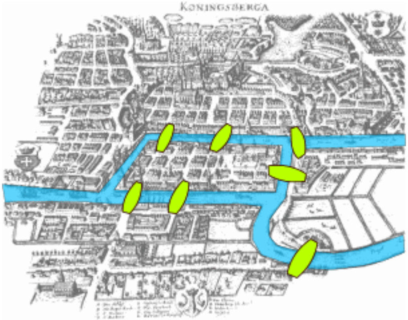

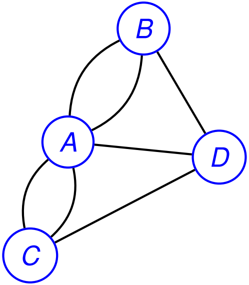

Can you draw a tour in the graph where you visit each edge once?
Yes?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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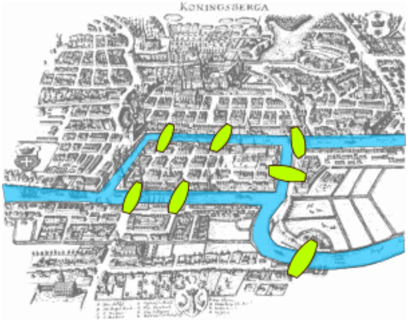

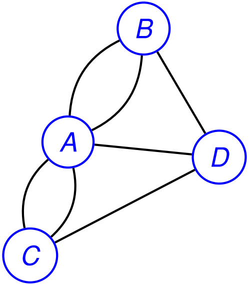

Can you draw a tour in the graph where you visit each edge once?
Yes? No?

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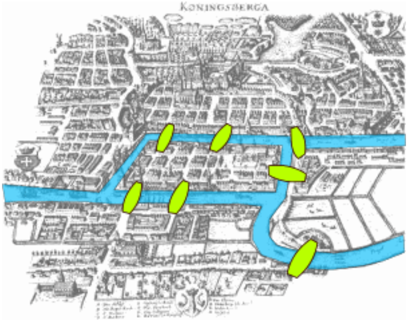

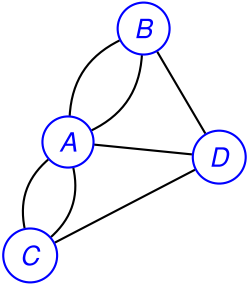

Can you draw a tour in the graph where you visit each edge once?
Yes? No?
We will see!

Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

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Theorem: Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

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Eulerian Tour is connected so graph is connected.

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Tour enters and leaves vertex v on each visit.

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Tour enters and leaves vertex v on each visit.
Uses two incident edges per visit.

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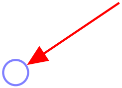

When you enter,

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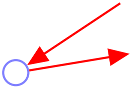

When you enter, you can leave.

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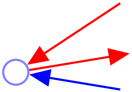

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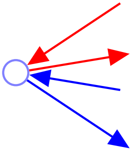

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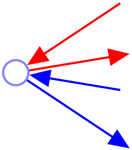

When you enter, you can leave.
For starting node,

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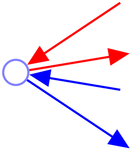

When you enter, you can leave.
For starting node, tour leaves first

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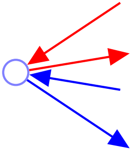

When you enter, you can leave.
For starting node, tour leaves first ....then enters at end.

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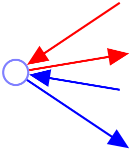

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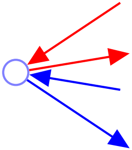

When you enter, you can leave.
For starting node, tour leaves first ....then enters at end.
Not The Hotel California.

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm.

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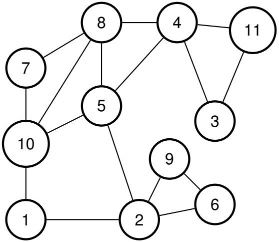
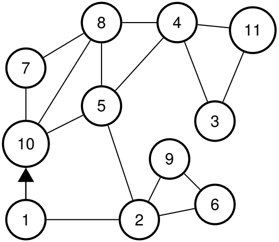


1. Take a walk starting from v (1) on “unused” edges

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.



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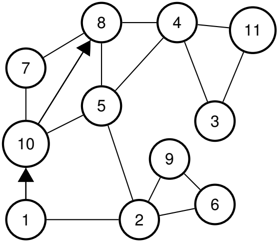

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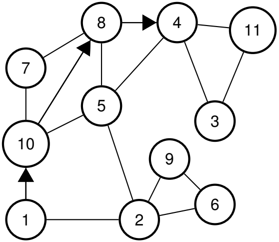

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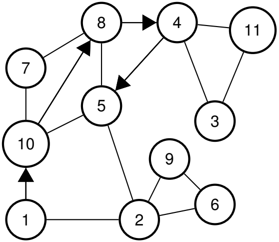

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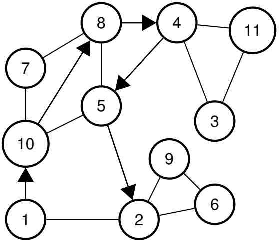

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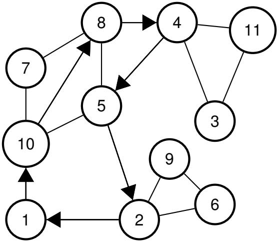

1. Take a walk starting from v (1) on "unused" edges
... till you get back to v.

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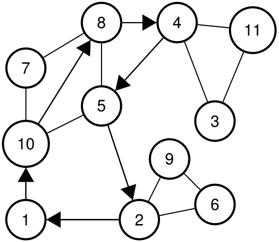


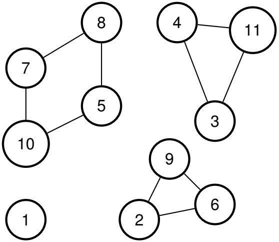
1. Take a walk starting from v (1) on "unused" edges

... till you get back to v.

2. Remove tour, C.

Finding a tour!

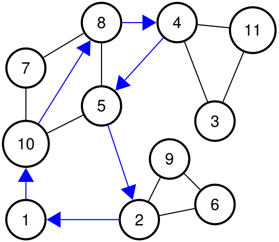
Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.



1. Take a walk starting from v (1) on "unused" edges
... till you get back to v.
2. Remove tour, C.
3. Let $G_1, \ldots, G_k$ be connected components.

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.



1. Take a walk starting from v (1) on "unused" edges
... till you get back to v.
2. Remove tour, C.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by C.

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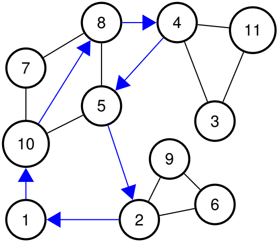

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      Why?

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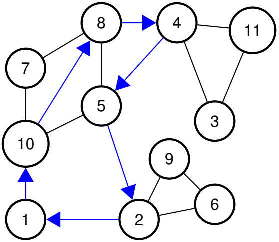

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        Why? G was connected.

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Proof of if: Even + connected $\Longrightarrow$ Eulerian Tour.
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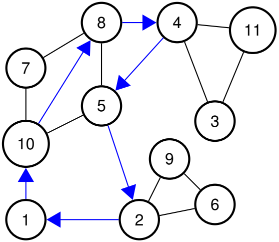

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   Each is touched by C.
      Why? G was connected.
   Let v_i be (first) node in G_i touched by C.

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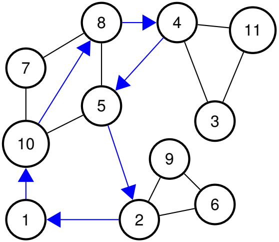

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   Let v_i be (first) node in G_i touched by C.
    Example: $v_1 = 1$,

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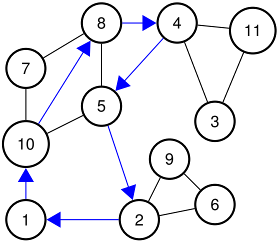

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   Each is touched by C.
      Why? G was connected.
   Let v_i be (first) node in G_i touched by C.
    Example: $v_1 = 1$, $v_2 = 10$,

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
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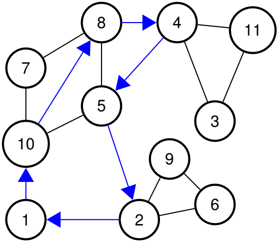

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    Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$,

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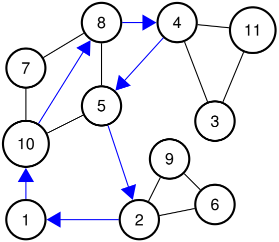

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    Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.

Finding a tour!

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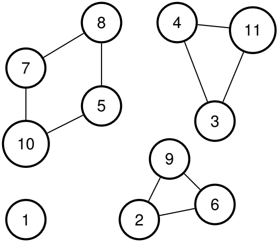

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   Let v_i be (first) node in G_i touched by C.
    Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from v_i

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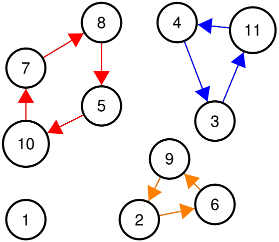

1. Take a walk starting from v (1) on "unused" edges
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3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by C.
      Why? G was connected.
   Let v_i be (first) node in G_i touched by C.
    Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
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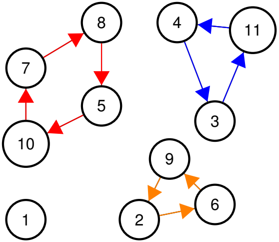

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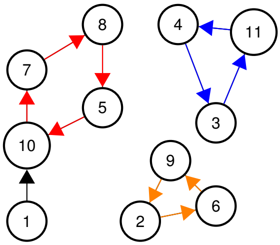

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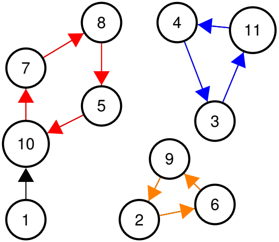

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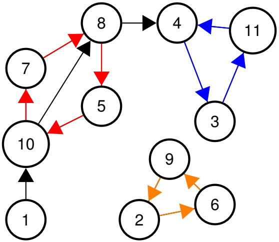

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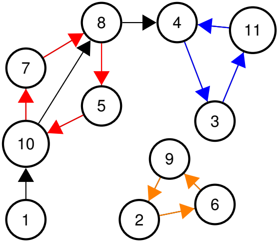

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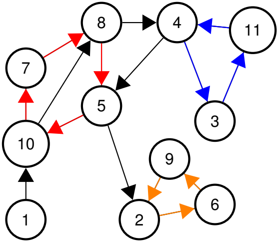

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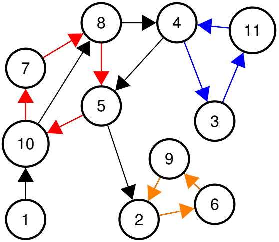

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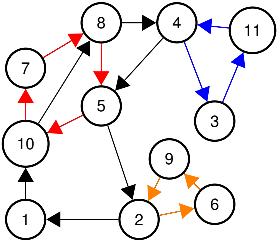

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Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

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Proof of Claim: Even degree.

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Claim: Each vertex in each G_i has even degree and is connected.

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3. Find tour T_i of G_i starting/ending at v_i. Induction.

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Visits every edge once:
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Poll: Euler concepts.

Mark correct statements for a connected graph where all vertices have even degree. (Below, we use tour to mean uses an edge exactly once, but may involve a vertex several times.

Poll: Euler concepts.

Mark correct statements for a connected graph where all vertices have even degree. (Below, we use tour to mean uses an edge exactly once, but may involve a vertex several times.

(A) Removing a tour leaves a graph of even degree.
(B) A tour connecting a set of connected components, each with a Eulerian tour is really cool! Eulerian even.
(C) There is no hotel california in this graph.
(D) After removing a set of edges E' in a connected graph, every connected component is incident to an edge in E'
(E) If one walks on new edges, starting at v, one must eventually get back to v.
(F) Removing a tour leaves a connected graph.

Poll: Euler concepts.

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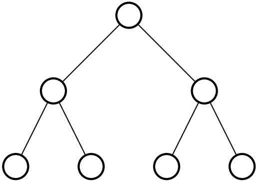
(F) Removing a tour leaves a connected graph.

Only (F) is false.

A Tree, a tree.

Graph $G = (V, E)$.

Binary Tree!



More generally.

Trees.

Definitions:

Trees.

Definitions:

A connected graph without a cycle.

Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V| - 1$ edges.

Trees.

Definitions:

A connected graph without a cycle.
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A connected graph where any edge removal disconnects it.

Trees.

Definitions:

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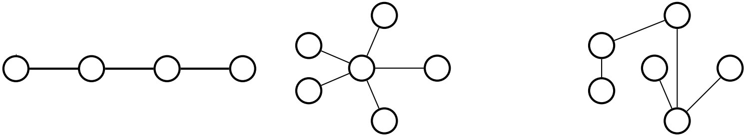
Some trees.



no cycle and connected?

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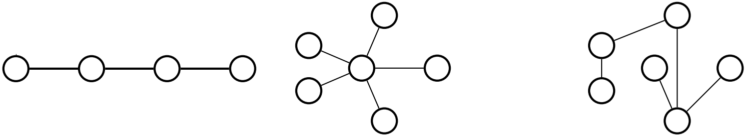

no cycle and connected? Yes.

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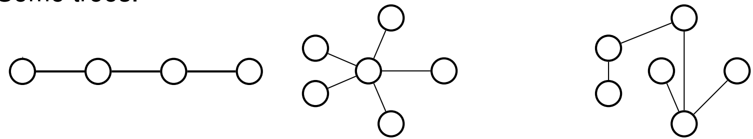

no cycle and connected? Yes.
$|V|-1$ edges and connected?

Trees.

Definitions:

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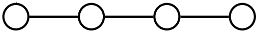
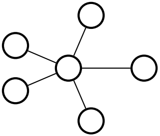
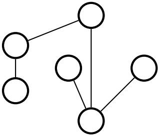

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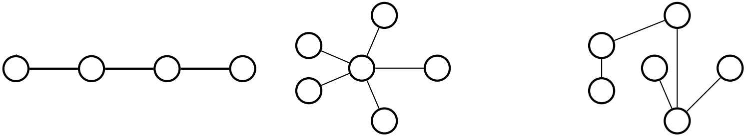

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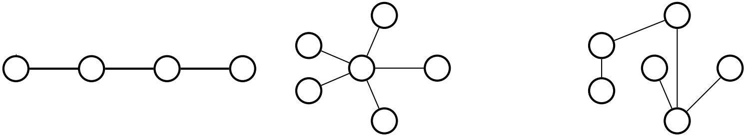

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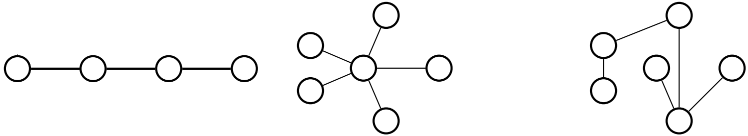

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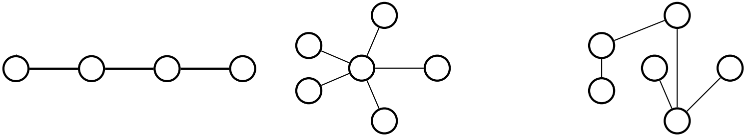

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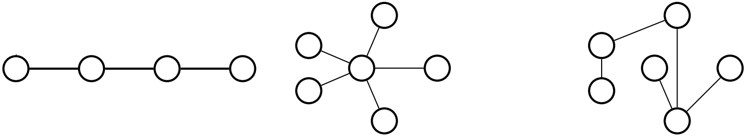

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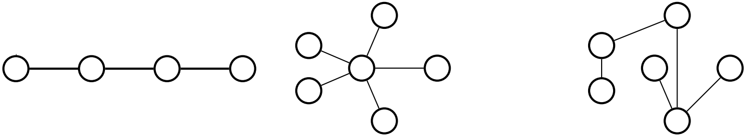

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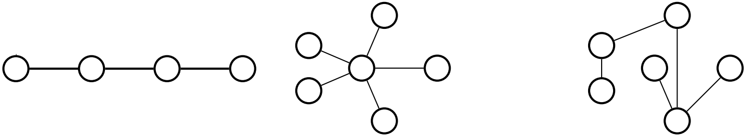

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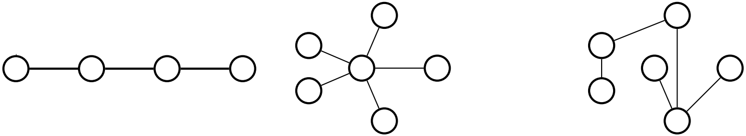

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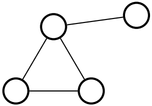

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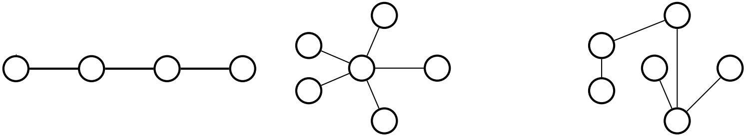

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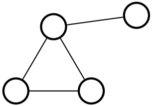

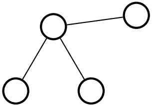

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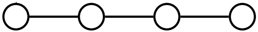
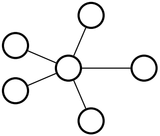
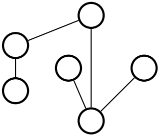

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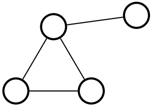
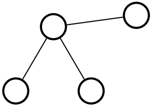
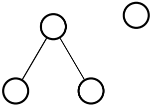

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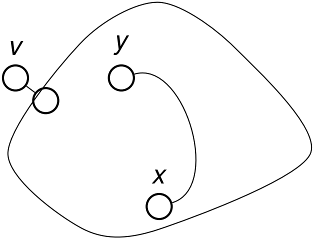

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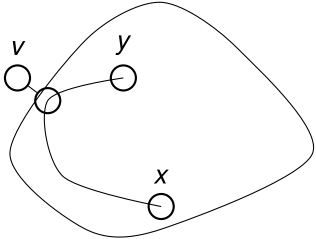

Proof of only if.

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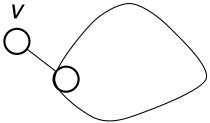

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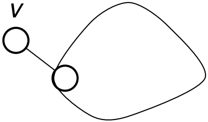

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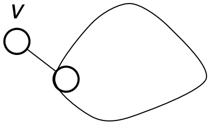

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"G connected and has $|V|-1$ edges" $\Longrightarrow$
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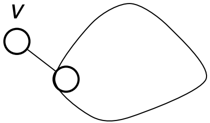

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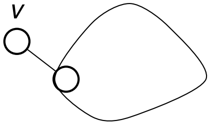

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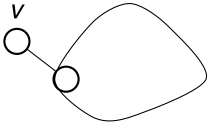

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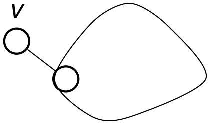

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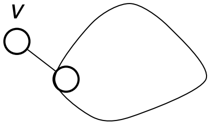

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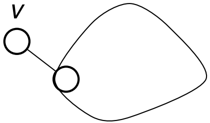

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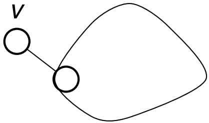

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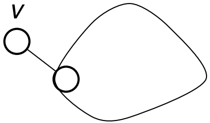

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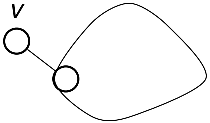

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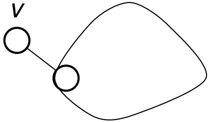

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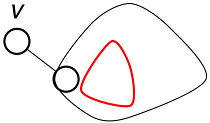

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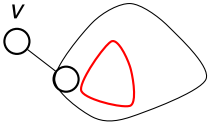

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And no cycle in G since degree 1 cannot participate in cycle.

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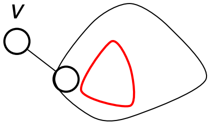

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And no cycle in G since degree 1 cannot participate in cycle. $\square$

Proof of if

Thm:
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
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Walk from a vertex using untraversed edges.

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Until get stuck.

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Walk from a vertex using untraversed edges.
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Claim: Degree 1 vertex.

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Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
Proof of Claim:
Can't visit more than once since no cycle.

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Thm:
"G is connected and has no cycles"
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Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
Proof of Claim:
Can't visit more than once since no cycle.
Entered.

Proof of if

Thm:
"G is connected and has no cycles"
$\implies$ "G connected and has $|V|-1$ edges"
Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
Proof of Claim:
Can't visit more than once since no cycle.
Entered. Didn't leave.

Proof of if

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Until get stuck.
Claim: Degree 1 vertex.
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Can't visit more than once since no cycle.
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Removing node doesn't create cycle.

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New graph is connected.

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Removing degree 1 node doesn't disconnect from Degree 1 lemma.

Proof of if

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By induction $G-v$ has $|V|-2$ edges.

Proof of if

Thm:
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
Proof of Claim:
Can't visit more than once since no cycle.
Entered. Didn't leave. Only one incident edge. $\square$
Removing node doesn't create cycle.
New graph is connected.
Removing degree 1 node doesn't disconnect from Degree 1 lemma.
By induction $G - v$ has $|V| - 2$ edges.
G has one more or $|V| - 1$ edges.

Proof of if

Thm:
"G is connected and has no cycles"
$\implies$ "G connected and has $|V|-1$ edges"
Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
Proof of Claim:
Can't visit more than once since no cycle.
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Removing node doesn't create cycle.
New graph is connected.
Removing degree 1 node doesn't disconnect from Degree 1 lemma.
By induction $G-v$ has $|V|-2$ edges.
G has one more or $|V|-1$ edges. $\square$

Poll: Oh tree, beautiful tree.

Let G be a connected graph with $|V| - 1$ edges.

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Let G be a connected graph with $|V|-1$ edges.

(A) Removing a degree 1 vertex can disconnect the graph.
(B) One can use induction on smaller objects.
(C) The average degree is $2-2/|V|$.
(D) There is a hotel california: a degree 1 vertex.
(E) Everyone can be bigger than average.

Poll: Oh tree, beautiful tree.

Let G be a connected graph with $|V| - 1$ edges.

(A) Removing a degree 1 vertex can disconnect the graph.
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(D) There is a hotel california: a degree 1 vertex.
(E) Everyone can be bigger than average.

(B), (C), (D) are true

Lecture in a minute.

Graphs.

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Graphs.
  Basics.

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Graphs.
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Connectivity.

Lecture in a minute.

Graphs.
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Connectivity.
Algorithm for Eulerian Tour.

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Trees: degree 1 lemma $\implies$ several definitions.

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Planar Graphs: intro.